Published as:
Sellés-Martínez, J., 2007. Lze popsat krásu matematickým výrazem? *en* Umela inteligence, Editores V. Marik, O. Stepankova y J. Lazansky. National Academy of Science (Praga), págs. 173-184.

CAN BEAUTY BE REDUCED TO A MATHEMATICAL EXPRESSION?

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ABSTRACT

Although several forms of beauty can be described in mathematical language, beauty is an open concept which meaning has continuously changed with Ages, geographical location and several other more subtle factors -like social perception of “good and bad taste”- precluding any definitive definition. The relationship between Mathematics and Art has been, nevertheless; very fruitful, the Golden Rule has set the standards for perfection (one of the volatile attributes of beauty) for centuries. Geometrical perspective, tessellation or fractal art, along with several others, are discussed and illustrated in this contribution, taking in account that these mathematical approaches to design can be a seen as the first steps in the way to the implementation of virtual emulators of Leonardo da Vinci.

Keywords: Science and Art, Mathematics, the Golden Rule, Proportion, Fractals

INTRODUCTION

Can beauty be reduced to a mathematical expression? The answer to this question may be not unique. "Philosophers who believe that there is no beauty in Mathematics are wrong. Mathematics is indeed the supreme form of Beauty", says Aristotle. "Art begins where calculations end", argues Le Corbusier, tracing a neat border between both disciplines.

May the difference relay in the intention of the author? A line that can be described by a mathematical expression, a form that can be described by spatial geometry; are only Mathematics when drawn on the blackboard during a class but Art when offered to the public on a canvas?. Figure 1a reproduces a picture found in Internet and offered as a work of art; on the other hand Figures 1b and 1c have been taken from a book on Structural Geology (Ragan, 1984) and represent models for superimposed folding on sedimentary strata. It comes out easily that anyone colouring the pattern in Figure 1c would be able to produce his own work of art from a geometrical pattern representing the results of a geological process.

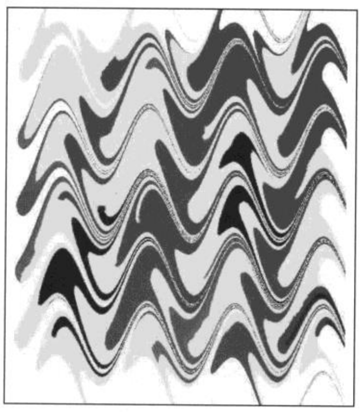
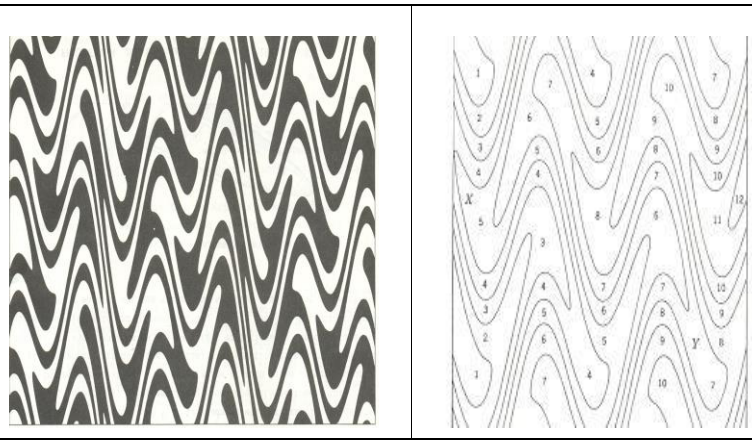
	

Figure 1a	Figure 1b and c

Far away from having an easy and unique answer, the questions issued above give birth to multiple other ones, like "What is Art?" or "What is Beauty?" or "What does **perfect** mean in relation to Beauty?" The answers have changed along time in the same culture, and have also been different at the same time in different cultures. The Venus of Milo, after being an archetype of beauty during centuries in western countries, is almost ignored by today's fashion. The artistic production of Africa has only been appreciated in the XXth Century; before then, it was more of anthropological interest than of artistic value.

Going further, the use of mathematical tools to improve or even generate images that are presented as works of art and to the idea that machines can produce art by themselves.

The aim of this contribution is not offering a definitive answer to the above mentioned problems, but to contribute to the discussion, analyzing the most important inputs to Art from Mathematics, but without forgetting that for each example of the application of Mathematics in a work of Art, there is a counter-example of a work of Art that is in anyway related to them.

PROPORTION AND THE GOLDEN RULE: HOW CAN A SEGMENT BE BEAUTIFULY DIVIDED

As long as a point is dimensionless, no problem arises when trying to divide it, it is indivisible. When dealing with a line, and taking in account that its length is infinite, it does not matter where we place a point to divide it into two; both parts will be equally long. Trouble arises when trying to divide a segment into two parts. Which is the best point to place the division?

For many centuries, from the Egyptians on, the answer has been "Place it at the point where the ratio of the minor segment to the major is the same as the ratio of the major to the whole". This statement, illustrated in Figure 2a, is in the bases of the development of the Golden Rule, which has been extended from the line to the plane and from it to the space, leading to the Golden Rectangle, Golden Polyhedrons, Golden Spirals, etc. More over, the Golden rule has been extended to control proportionality between anything in which more than one part could be present, including the human body, the plan of a cathedral, the scale of colours, the scale of music, and so on.

The numerical expression of this canon for proportionality leads to a series of numbers know as "The series of Fibonacci", named after the so-called Italian mathematician of the Medieval Age, Leonardo da Pisa. The ratio of any two consecutive numbers in the series tends to the value 1,618, the *Golden Number*. As stated above, this Golden Number has long and intensively used to rule the design of buildings, sculptures, drawings and so on.

All classical art in western civilization is, in one way or another, influenced by the Golden Number. Figure 2b illustrates how a sculptor can design a human body using the Golden Number to find the relative length of all its components.

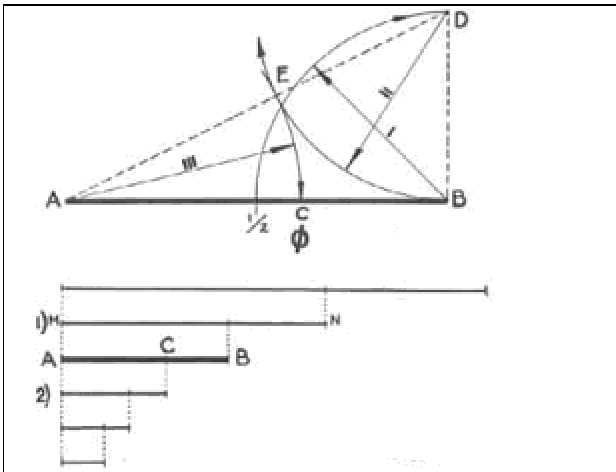


Figure 2a: Division of a segment into two proportional ones according to the Golden Rule (From Tosto, 1969, Fig. 1).

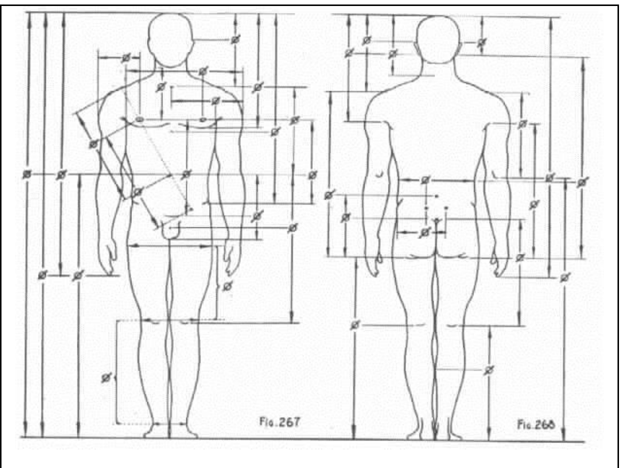


Figure 2b: Sketches describing the proportion of a perfect male body according to the Golden Rule, (From Tosto, 1969, figure 267).

Painters like El Greco, Durero or, most recently, Modigliani, or Botero, do not follow the Golden Rule when designing their works. Do we have to think that they are not masters, or that they are of an inferior rank than those that follow the rule? What about those portraits and still-lives from the masters of cubism? Are they not as *perfect* as those Italian or Dutch classic masterpieces “done by the book” of the Golden Rule?

PATTERNS IN PLANE AND SPACE: MONOTONY AND RHYTHM

Tessellation is the field of Mathematics that deals with the partition of surfaces. Tessels are small, flat pieces of stone or tile that can be organized in many different ways to cover a surface. Mosaics from all ages testify their importance for architects and designers. Vitraux may sometimes be regarded as a particular type of mosaic, in which tessels are pieces of colour glass. Carpets and rugs may also be regarded as knitted or embroidered mosaics.

It is not difficult to find out that tessellation principles and mosaic design have had a long and fruitful relationship throughout styles. What seems astonishing is that, no matter the number of designs may seem infinite; they all come out from a few basic patterns. In the case of a flat surface, the number of basic grids is only four. The square, rectangular, rhomboidal or hexagonal grids, illustrated in figure 3 give the basis to all known designs, while figure 3b further illustrates the application.

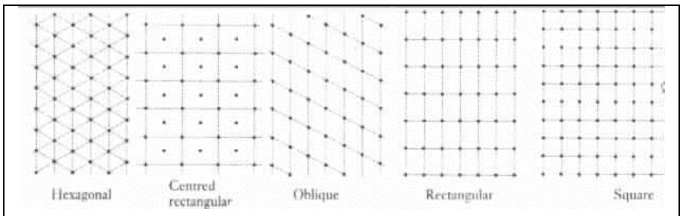


Figure 3: Basic grids for regular designs on a flat surface (from Mercer, 1990).

The designs in figure 4a illustrate the seventeen basic patterns, most of them surprisingly common to different cultures since very ancient times. The Islamic art, lacking the chance to reproduce human or animal beings, mastered geometrical design. Figure 4b illustrates how a basic grid may be combined with a highly complex design to produce a regular pattern. Addition of colour produces an explosion of fascinating designs.

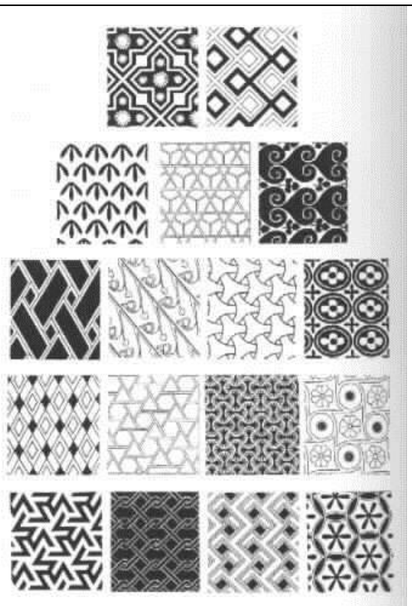


Figure 4a: The seventeen basic archetypes of designs in two dimensions (from P. Stevens, 1981, in Strossberg, 2001)

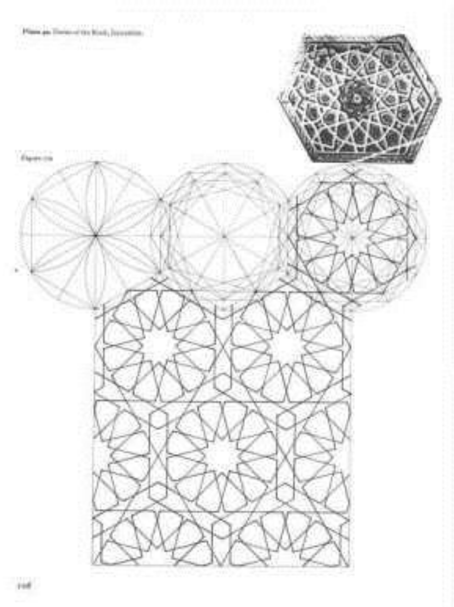


Figure 4b: The building of a design for the palace of Alhambra (from El Said and Parman, 1976)

SYMMETRY IN PLANE AND SPACE: FROM MIRRORS TO CRYSTALS

"Symmetry is the beauty resulting from right relations of parts, the quality of harmony or agreement (in size, design, etc.) between parts", reads the definition in many dictionaries. Most common form of symmetry is the presence of a line dividing two fields, each one being the mirror image of the other. More complex forms of symmetry include the *axis*, which repeats a shape *n* times along a 360º rotation; the *plane* which produces a mirror image in the space; and the *centre* of symmetry, which repeats any feature in the opposite side of a 3D body, combining rotation and inversion. Planes, axis and centres of symmetry are well known to crystallographers. Figure 5a illustrates the elements of spatial symmetry used to describe the shapes of crystals. Figure 5b represents a painting with an axis of symmetry repeating the same pattern twice.

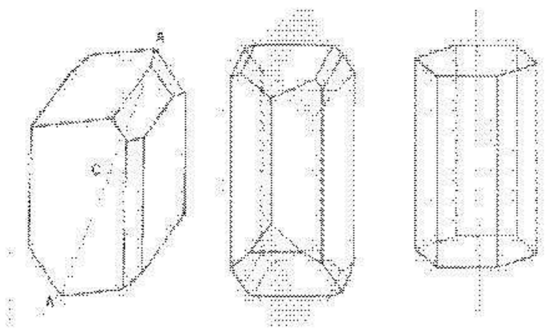


Figure 5a: Centre, plane and axis of symmetry.

Figure 5b: Australian contemporary painting in traditional style.

The search for perfect polyhedrons (or "Platonic bodies") has rendered poor results, there are five and only five of them, the ones illustrated in Figure 6a. Nowadays interest in regular (not only platonic) bodies is centred in the study of three dimensional arrays of atoms in natural and technological materials, being their physical properties a direct consequence of how their constituent atoms are ordered. Surprisingly, only a few different arrays of atoms, ions or molecules are possible, Figure 6b shows the 14 basic lattices (the so called “lattices of Bravais”, after the crystallographer that first described them) that give the bases to the six different crystalline systems that group the 230 possible 3D crystal lattices.

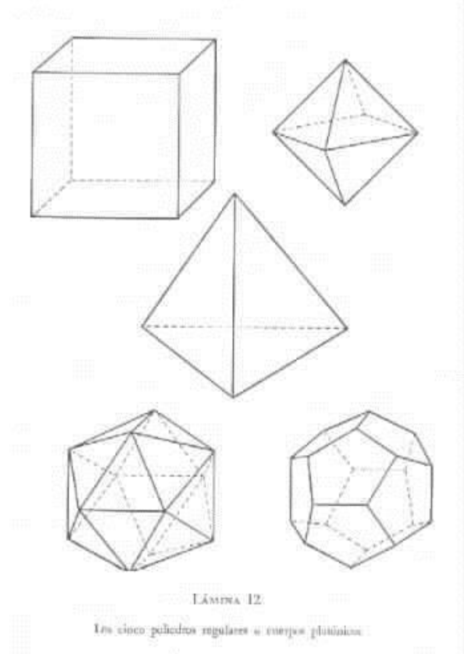


Figure 6a: The Platonic Bodies or perfect polyhedrons (from Ghyka, 1983, Plate 12)

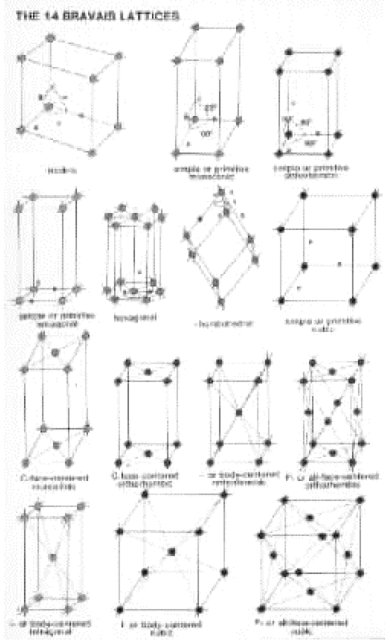


Figure 6b: The 14 basic crystalline lattices (from Mottana et al., 1996, p 24).

Surpassing the static designs of mosaics and crystals and also pushing forward the work of M.C. Escher calidocycles make their appearance. They are rings of tetrahedrons giving a 3D dynamic dimensionality to Escher´s most famous geometrical tricks, as illustrated in Figure 7. The design in each face of a calidocycle combines with the design in the neighbouring ones. But the pattern changes when rotating the calidocycle!, making and endless rhythm of combining designs.

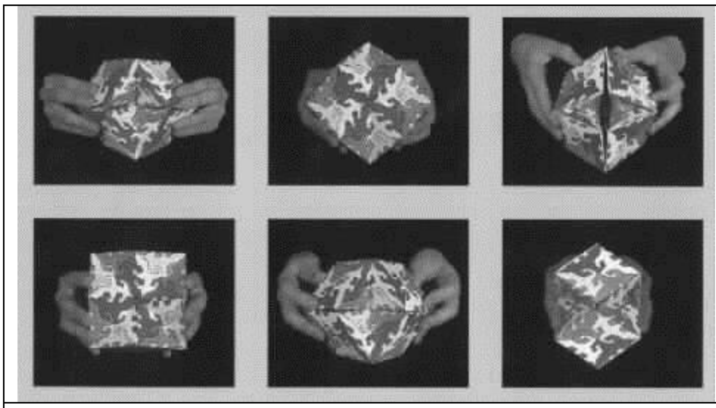


Figure 7: A calidocycle designed after an Escher´s drawing (From Schattschneider and Walker, 1992, back cover).

PERSPECTIVES AND PROJECTIONS: CAPTURING THE THIRD DIMENTION

The development of almost perfect methods of perspective and projections has been based on geometry. The needs of cartographers fuelled the development of the many types of projections that help representing the sphere of the Earth in the plane, whilst the need to produce the illusion of depth in a two dimensional picture helped developing the rules of perspective.
Outstanding examples of the use of perspective and projection benchmark the history of Science and Art. Figures 8a and b, are two among thousands of examples of the use of geometrical perspective and maps (a type of projection) in Art.

Figure 8a: Antonello da Messina

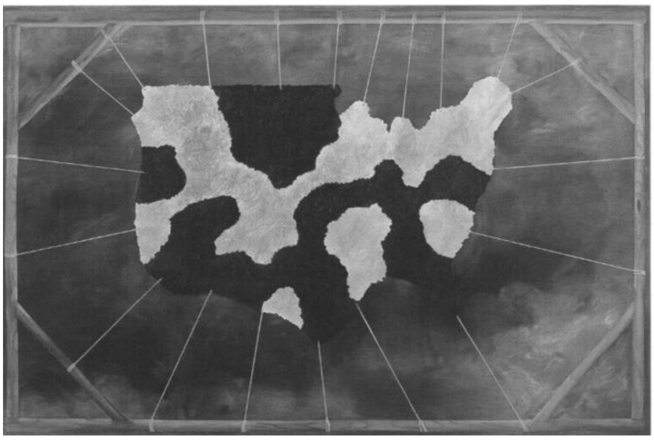
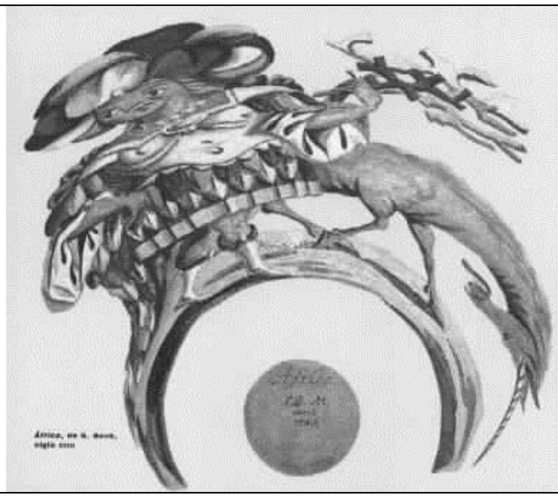


Figure 8b: Arizona, Carlos Larrondo

Although nowadays satellites take images of the Earth from space, making the work of map drawers much easy than never before, these images still have to be modified in order to adequate them to a conventional projection and to make it useful for cartographical purposes. On the other hand, painters have long left away the need of perspective and have explored simultaneous multiple sights of the same object (as in cubism) or definitively abandoned the idea (or the need) of representing the third dimension. In the way to this point, some interesting achievements may be underlined, like the *anamorphosis* or exaggerated distortion of the shape resulting from a very oblique point of view (an example from Holbein being illustrated in Figure 9a) or, even more complex and requiring cylindrical mirrors to be reconverted into a decipherable image the example in Figure 9b, designed by E. Beck.

	
Figure 9a: The skull is only seen in its real shape when looking at the canvas from its left and down corner, whilst the rest of the table has been designed to be seen from the front (picture from Hans Holbein depicting an *Ambassador*).	*Figure 9b*: A cylindrical mirror is required to reconstruct this fan shaped image painted by E. Beck, representing *Africa*.

Striking pseudo perspectives, like the one from Escher reproduced in Figure 10 must be considered a proof of master skill.

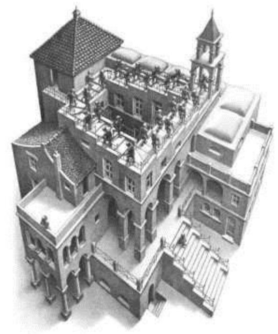
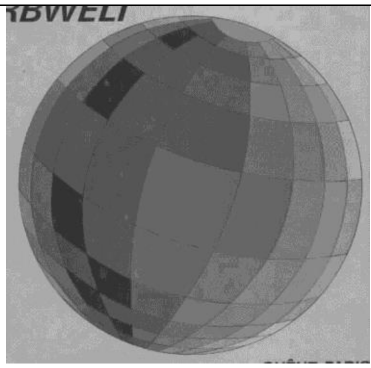


Figure 10a: This never-ending staircase will be never be built in three dimensions, no matter it has been etched by Escher on a two dimensional surface.

The art of projection has fascinated painters, sometimes with obvious references to the spherical shape of the Earth (as in Figures 11a and 11b) or, going back and further in projecting, representing a planar image of an spherical distortion of what originally was a flat design (as in Figures 12a and 12b).

	
Figure 11a: In this engraving from Escher (*Ball spires*), lines are still clear references to parallels, meridians and loxodromic lines.	*Figure 11b*: This design from Vasarely, named *Planet MC* is also still strongly related to the latitude-longitude grid.

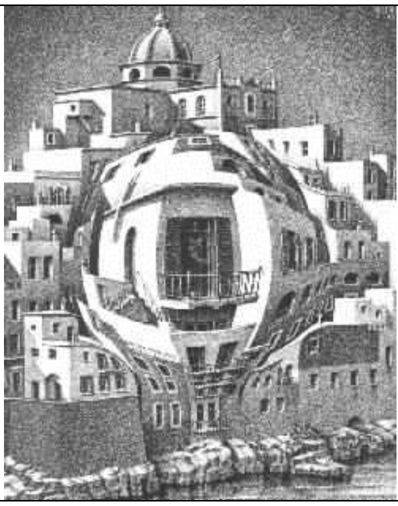


Figure 12a: Spherical distortion of a plane. A view of a *Balcony* from Escher.

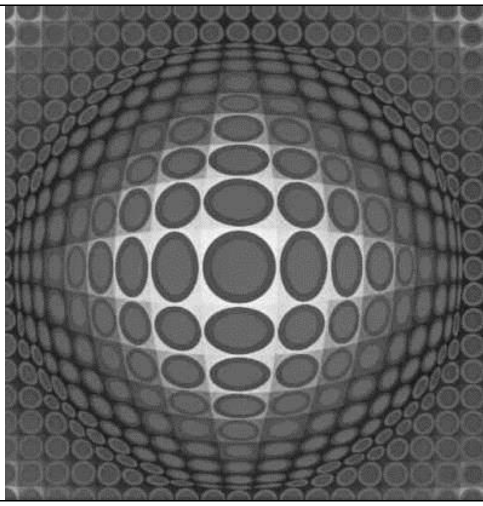


Figure 12b: Spherical distortion of a plane. *Vega-tek* from Vasarely.

GEOMETRICAL AND FRACTAL ART

Most of the examples given above illustrate the fruitful relationships of Art and Mathematics, but none of them has been done with the only purpose of presenting mathematics as a fine art. Fractal art, rendering attractive patterns out of formulas, may be the exception; it has been used to produce appealing designs that have even been sold for decorative purposes.

Figure 13a and Figure 13b show two different fractal designs, the first one is essentially aesthetic, the other represents physical phenomena.

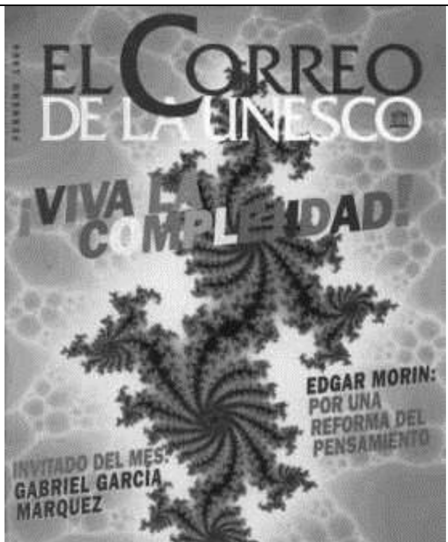


Figure 13a: A fractal design from El Correo de la Unesco (February, 1996, front cover).

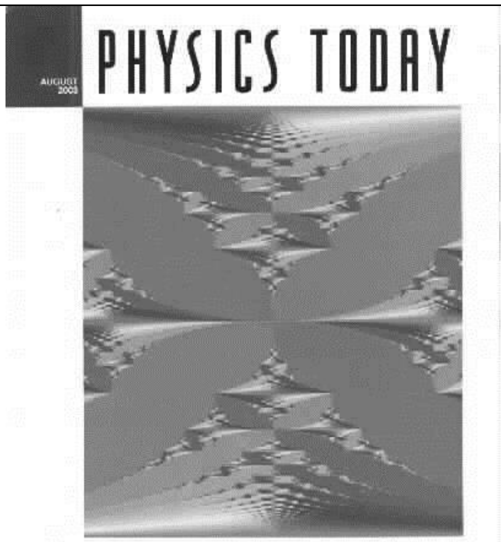


Figure 13b: The Hofststadter´s Butterfly from Physics Today (August 2003, V 56 N 8, front cover)

During last years, fractal design has developed itself into a plastic form of expression, and many artists exhibit their works. An example of that is a recent exhibition in Madrid, honouring Dr. Mandelbrott during the 2006 International Mathematical Congress, from which Figures 14a and 14b have been taken. Several mathematicians think that this is not Mathematics; many artists think that is not Art. Who is right?

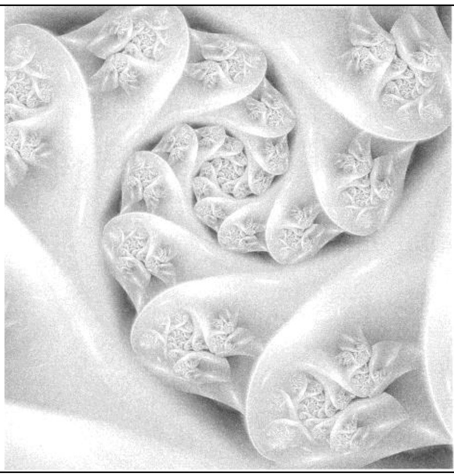


Figure 14a

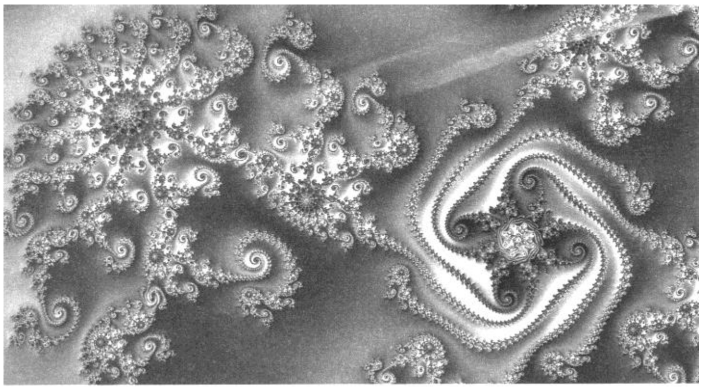


Figure 14b

Interestingly indeed are the two masterpieces in Figures 15a and 15b. One is the well known "The Great Wave" from Hokusai, but the other is not so famous, no matter it incorporated fractals to the history of art as long ago as the XVth Century, long before mathematicians discovered them.

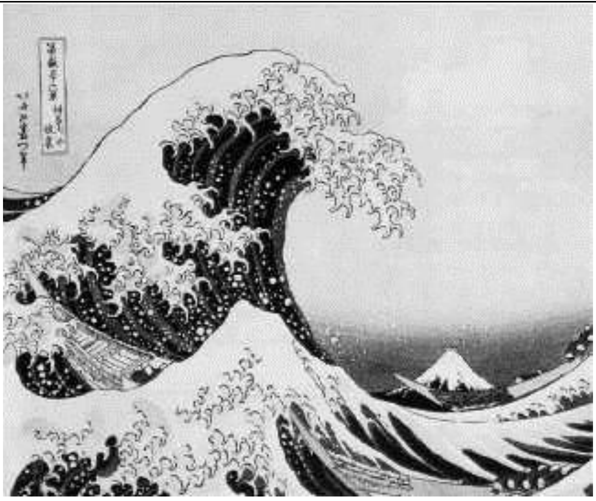


Figure 15a: Fractals in art. *The Great Wave* from Hokusai.

Figure 15b: Fractal bushes in a painting by Domenico Veneziano.

A quite more sophisticated Fractal relationship is found in antique flower patterns from Chinese pottery (but not only there), in which the amount of flowers or elements of different sizes follows a certain fractal relationship.

CONCLUSIONS

A beautifully bended line, like a meander, may be easily described by a mathematical formula. But this does not make the line the product of a mathematical formula. The design predates the formula that describes it. This is clearly illustrated by the observation of such designs in the decorative arts of peoples that had no mathematical knowledge. On the other side, machines can be loaded with the necessary software to automatically produce attractive designs and even more, to produce 3D images and animations. But the product of the application of an algorithm or the result of the manipulation of a formula in the context of a computer program is not Art created by the machine, as popularly thought. Somebody who uses a computer in stead of a pencil, a chisel or brushes is hidden behind the mouse, so to say; as long as the author is aware, no machine is still able to really create new shapes without previous instructions given by a man (not to mention that machines still need somebody to switch them “on” before anything happens). Artificial geniality has not yet been achieved and this keeps the space exclusively open to human beings (and may be to several animals…). Let the artist continue using every tool available to him, from the pieces of coal of primitive men to the most sophisticated computer program of today. What will come out of his inspiration will be Art no matter it has been based in his mathematical knowledge or not.

AKNOWLEDGEMENTS
The author is grateful to the organizers of the E-Golem for their kind invitation to participate in the meeting.

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