Under pressure
Calculating the intense pressures underground

This Earthlearningidea has been devised to give an idea of the intense pressures underground through measurement and calculation. It is presented in three systems of units on the following pages.

In all cases Newton's equation of:
force = mass x acceleration
remains true and the acceleration term has been included throughout

(1) SI system
This system is in common use in secondary schools in the UK. Measurements are made in the secondary units millimetres and grammes and results presented in the primary units, Newtons per square metre. This unit is also known as the Pascal (Pa). The numeric value of the acceleration due to gravity, g is 10 m/sec^2.

Photo: S. Allen and G. Jones

(2) gramme-centimetre (cgs) system
This system may be easier for school pupils to visualise the units, as they commonly measure in centimetres. It is customary to define the acceleration due to gravity 'a' as:
a = 1 x earth gravity (g) rather than
a = 981 cm/sec^2
so that grammes (mass) and grammes (force) have the same numeric value, though the dimensions differ. Here, measurements are made in grammes and centimetres and the results presented in the same units.

(3) Imperial system
As above, pounds (force) and pounds (mass) are numerically equal; implying that a = 1 x earth gravity (g) rather than a = 32 feet/second^2.

Absolute units of force do exist in the cgs and Imperial systems – the dyne and the poundal respectively – but they are seldom used. They would be used for calculations where the local gravity differs from that of the Earth.

Under pressure
Calculating the intense pressures underground – in SI units

Pressure is defined as force per unit area. We can use this to get a good idea of the intense pressures underground through measurement and calculation, by using the equation:

$$\text{Pressure in N/m}^2 \text{ (Pa)} = \frac{\text{mass in kg x 10}}{\text{area in m}^2}$$

1. Rock pressure underground

It is difficult to cut rocks into the right size and shape to do pressure measurements and calculations, so we use loose sand instead. The calculations give a realistic impression of lithospheric pressures at different depths (i.e. the pressure of the overlying rocks on the rocks beneath – the overburden pressure). At depth, sand becomes sandstone due to cementation and compaction.

a) Calculate the mean mass of 10mm thickness of sand

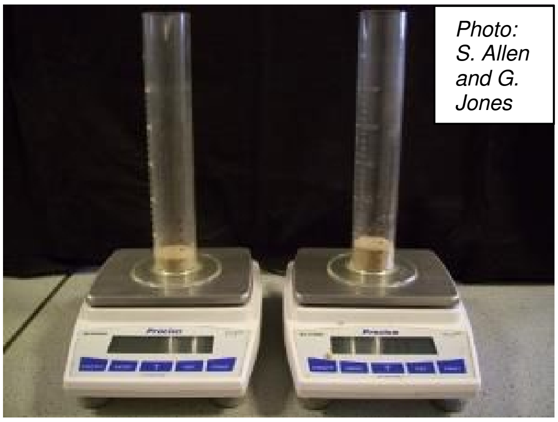

Photo: S. Allen and G. Jones

- Use a pen to mark a measuring cylinder with 10mm intervals, from the bottom up.
- Add 10mm depth of sand to the measuring cylinder; tap it to ensure it is level, then weigh it.
- Add another 10mm depth of sand, tap it and weigh again.
- Subtract the first figure from the second to find the mass of the second 10mm depth sand – then note down the result.
- Repeat this several times.
- Add together the results of all the mass calculations (ignoring the mass of the first 10mm depth of sand – which is unreliable) and divide by their number to find the mean mass of 10mm depth of sand.
- (*For a normal 200ml measuring cylinder and normal sand, the mass of 10mm thickness of sand is around 23g or 0.023kg*).

b) Calculate the area of the cylinder

- Measure the diameter of the inside of the measuring cylinder.
- Divide the measurement in half to give the radius of the internal base of the cylinder.
- Use the formula below to calculate the area of the base of the cylinder.

Area of a circle = πr^2
= π (3.142) x radius in m x radius in m

(*A 200 ml normal plastic measuring cylinder has an internal diameter of 40mm or 0.04m, a radius of 20mm or 0.02m and a base area of 3.142 x 0.02 x 0.02 = 0.00126m²*).

c) Calculate the pressure of 10mm sand

- Use the equation opposite to calculate the pressure of 10mm depth of sand.

(Pressure = $\frac{m \times a}{A} = \frac{0.023 \times 10}{0.00126 m^2} = 183 \text{ N/m}^2$

This is the equivalent of 183gm on the area of approximately a thumbnail).

d) Calculate the pressure of 1m, 100 m, 1 km, 10 km, and 100 km of sand

- Use these calculated figures to work out the pressures at the depths shown in the table below.

Table for use in part d).

Depth	Depth equivalent	Multiply figure above by:	*Pressure using figures above*	*Pressure equivalent* (thumbnail approx.0.01m²)
1m	Table to the floor	100	*183x 100 = 18,300 N/m²*	*183gm on a thumbnail*
100m	Short borehole	100	*1,830,000 N/m² (1.8MPa)*	*183 kg on a thumbnail (18 bags of sugar)*
1 km	Longer borehole	10	*18,300,000 N/m² (18MPa)*	*1,830 kg or 1.8 tonnes on a thumbnail*
10 km	Deepest boreholes ever drilled; halfway through the mean thickness of the crust	10	*183,000,000 N/m² (180MPa)*	*1.83 tonnes on a thumbnail*
100 km	Thickest parts of crust – under mountain chains	10	*1,830,000,000 N/m² (1.83 GPa)*	*18.3 tonnes on a thumbnail*

The actual pressures would be greater than these calculations for loose sand suggest, because pressures from the rocks above cause compaction of sands which (together with cementation) forms them into sedimentary rocks; these are more dense than loose sands. The loose sand used here had a relative density of 1.8 whilst the relative density of sandstone is around 2.7, nearly 50% greater.

2. Water pressure underground
Repeat the activity using water instead of sand to calculate approximate hydrostatic pressures underground. This shows the sorts of pressures found in aquifers (the porous and permeable rock bodies from which groundwater is extracted). Most aquifers are less than 1 km beneath the surface.

(*For a 200ml measuring cylinder, 10mm depth of water has a mean mass of 12.6g. Using this figure and changing to N and m*

$$\text{Pressure} = \frac{m \times a}{A} = \frac{0.0126 \times 10}{0.00126\,m^2} = 100\,N/m^2\ )$$

1. **Total underground pressures**
The total vertical pressure underground is the lithostatic pressure and hydrostatic pressure added together since these both act together (with the atmospheric pressure added on top).

Table for water

Depth	Depth equivalent	Multiply figure above by:	Pressure using figures above
1m	Table to the floor	100	*100x 100 = 10,000N/m²*
100m	Short borehole	100	*1,000,000 N/m² (1.0 MPa)*
1 km	Longer borehole	10	*10,000,000 N/m² (10 MPa)*

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Under pressure
Calculating the intense pressures underground – in g/cm² (cgs system)

Pressure is defined as force per unit area. We can use this to get a good idea of the intense pressures underground through measurement and calculation, by using the equation:

$$\text{Pressure in grammes/cm}^2 = \frac{\text{mass, grammes}}{\text{area in cm}^2} \times 1$$

1. Rock pressure underground
It is difficult to cut rocks into the right size and shape to do pressure measurements and calculations, so we use loose sand instead. The calculations give a realistic impression of lithospheric pressures at different depths (i.e. the pressure of the overlying rocks on the rocks beneath – the overburden pressure). At depth, sand becomes sandstone due to cementation and compaction.

a) Calculate the mean mass of 1 cm thickness of sand

Photo: S. Allen and G. Jones

- Use a pen to mark a measuring cylinder with 1 cm intervals, from the bottom up.
- Add 1cm depth of sand to the measuring cylinder' tap it to ensure it is level, then weigh it.
- Add another 1 cm depth of sand, tap it and weigh again.
- Subtract the first figure from the second to find the mass of the second 1cm depth sand – then note down the result.
- Repeat this several times.
- Add together the results of all the mass calculations (ignoring the mass of the first 1 cm depth of sand – which is unreliable) and divide by their number to find the mean mass of 1 cm depth of sand.
  (*For a normal 200ml measuring cylinder and normal sand, the mass of 1 cm thickness of sand is around 23g*).

b) Calculate the area of the cylinder
- Measure the diameter of the inside of the measuring cylinder.
- Divide the measurement in half to give the radius of the internal base of the cylinder.
- Use the formula below to calculate the area of the base of the cylinder.

  Area of a circle = πr^2
  = π (3.142) x radius in cm x radius in cm

  (*A 200 ml normal plastic measuring cylinder has an internal diameter of 4 cm, a radius of 2 cm and a base area of 3.142 x 2 x 2 = 12.6 cm²*).

c) Calculate the pressure of 1 cm sand
- Use the equation opposite to calculate the pressure of 1 cm depth of sand.

  (*For a 200ml measuring cylinder, 1 cm depth of sand has a mean mass of around 23 gm. Using this figure:*

  $$\text{Pressure} = \frac{m \times g}{A} = \frac{23\text{gm} \times 1}{12.6\text{ cm}^2} = 1.83\text{ gm/cm}^2$$

  This is 1.83g on the area of approximately a thumbnail and is also 183 kg m^{-2} and 320Pa).

d) Calculate the pressure of 1m, 100 m, 1 km, 10 km, and 100 km of sand
- Use these calculated figures to work out the pressures at the depths shown in the table below.

Table for use in part d)

Depth	Depth equivalent	Multiply figure above by:	*Pressure using figures above*	*Pressure equivalent (thumbnail approx.1cm²)*
1m	Table to the floor	100	*1.83 x 100 = 183 gm/cm²*	*183 gm on a thumbnail*
100m	Short borehole	100	*18,300 gm/cm²*	*18 kg (18 bags of sugar) on a thumbnail*
1 km	Longer borehole	10	*183,000 gm/cm²*	*183 kg on a thumbnail*
10 km	Deepest boreholes ever drilled; halfway through the mean thickness of the crust	10	*1,830,000 gm/cm²*	*1.83 tonnes on a thumbnail*
100 km	Thickest parts of crust – under mountain chains	10	*18,300,000 gm/cm²*	*183 tonnes on a thumbnail*

The actual pressures would be greater than these calculations for loose sand suggest, because pressures from the rocks above cause compaction of sands which (together with cementation) forms them into sedimentary rocks; these are more dense than loose sands. The loose sand used here had a relative density of 1.8 whilst the relative density of sandstone is around 2.7, nearly 50% greater.

2. Water pressure underground
Repeat the activity using water instead of sand to calculate approximate hydrostatic pressures underground. This shows the sorts of pressures found in aquifers (the porous and permeable rock bodies from which groundwater is extracted). Most aquifers are less than 1 km beneath the surface.

(*For a 200ml measuring cylinder, 1 cm depth of water has a mean mass of 12.6g. Using this figure:*

$$\text{Pressure} = \frac{m \times g}{A} = \frac{12.6\text{gm} \times 1}{12.6\ \text{cm}^2} = 1.00\ \text{g/cm}^2)$$

3. Total underground pressures
The total vertical pressure underground is the lithostatic pressure and hydrostatic pressure added together since these both act together (with the atmospheric pressure added on top).

Table for water

Depth	Depth equivalent	Multiply figure above by:	Pressure using figures above
1m	Table to the floor	100	*1.0x 100 = 100 gm/cm²*
100m	Short borehole	100	*10,000 gm/cm²*
1 km	Longer borehole	10	*100,000 gm/cm²*

Under pressure
Calculating the intense pressures underground – in lb/sq in

Pressure is defined as force per unit area. We can use this to get a good idea of the intense pressures underground through measurement and calculation, by using the equation:

$$\text{Pressure in lb /sq in} = \frac{\text{mass in lb} \times 1}{\text{area in sq in}}$$

1. Rock pressure underground
It is difficult to cut rocks into the right size and shape to do pressure measurements and calculations, so we use loose sand instead. The calculations give a realistic impression of lithospheric pressures at different depths (i.e. the pressure of the overlying rocks on the rocks beneath – the overburden pressure). At depth, sand becomes sandstone due to cementation and compaction.

a) Calculate the mean mass of 1 inch thickness of sand

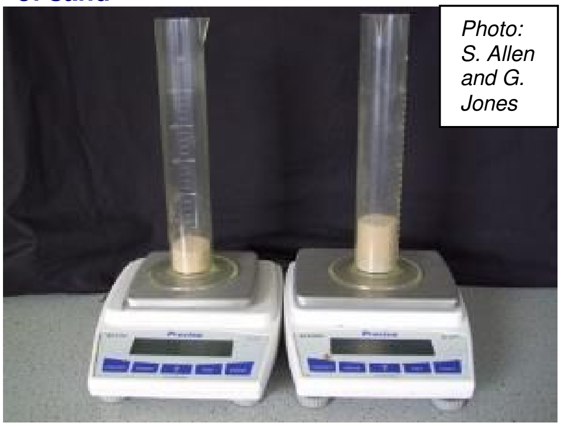

Photo: S. Allen and G. Jones

- Use a pen to mark a measuring cylinder with 1 inch intervals, from the bottom up.
- Add 1 inch depth of sand to the measuring cylinder' tap it to ensure it is level, then weigh it.
- Add another 1 inch depth of sand, tap it and weigh again.
- Subtract the first figure from the second to find the mass of the second 1 inch depth sand – then note down the result.
- Repeat this several times.
- Add together the results of all the mass calculations (ignoring the mass of the first 1 inch depth of sand – which is unreliable) and divide by their number to find the mean mass of 1 inch depth of sand.
  (*For a normal 200ml measuring cylinder and normal sand, the mass of 1 inch thickness of sand is around 0.129 lb*).

b) Calculate the area of the cylinder
- Measure the diameter of the inside of the measuring cylinder.
- Divide the measurement in half to give the radius of the internal base of the cylinder.
- Use the formula below to calculate the area of the base of the cylinder.

Area of a circle = πr^2
= π (3.142) x radius in inches x radius in in

(*A 200 ml normal plastic measuring cylinder has an internal diameter of 1.6 inches, a radius of 0.8 inches and a base area of 3.142 x 0.8 x 0.8 = 2.00 sq inches*).

c) Calculate the pressure of 1 inch of sand
- Use the equation opposite to calculate the pressure of 1 cm depth of sand.

$$\text{Pressure} = \frac{m \times g}{A} = \frac{0.129 \text{ lb} \times 1}{2.00 \text{ sq inches}} = 0.065 \text{ lb/sq inch}$$

This is 0.004 lb (1/6oz) on the area of approximately a thumbnail).

d) Calculate the pressure of 1 yard (3ft), 100 yards, 1 mile, 7 miles, and 70 miles of sand
- Use these calculated figures to work out the pressures at the depths shown in the table below.

Table for use in part d)

Depth	Depth equivalent	Multiply figure above by:	*Pressure using figures above*	*Pressure equivalent* (thumbnail approx.1cm^2)
1 yard	Table to the floor	36	*0.065 x 36 =2.34 lb/sq in*	*6 oz on a thumbnail*
100 yards	Short borehole	100	*234 lb/sq in*	*40 lb on a thumbnail 20 bags of sugar*
1 mile	Longer borehole	17.6	*234 x 17.6 = 4,118 lb/sq in*	*700 lb on a thumbnail*
7 miles	Deepest boreholes ever drilled; halfway through the mean thickness of the crust	7	*4,118 x 7 = 28,826 lb/sq in = 14.4 tons (US)/sq in*	*2.5 tons (US) on a thumbnail*

70 miles	Thickest parts of crust – under mountain chains	10	*288,260 lb/sq in (144 tons/sq in)*	*25 tons (US) on a thumbnail*

The actual pressures would be greater than these calculations for loose sand suggest, because pressures from the rocks above cause compaction of sands which (together with cementation) forms them into sedimentary rocks; these are more dense than loose sands. The loose sand used here had a relative density of 1.8 whilst the relative density of sandstone is around 2.7, nearly 50% greater.

2. Water pressure underground
Repeat the activity using water instead of sand to calculate approximate hydrostatic pressures underground. This shows the sorts of pressures found in aquifers (the porous and permeable rock bodies from which groundwater is extracted). Most aquifers are less than 1 km beneath the surface.

(*For a 200ml measuring cylinder, 1 inch depth of water has a mean mass of 0.072 lb. (1.16 oz) Using this figure:*

Pressure $= \frac{m \times g}{A} = \frac{0.072\text{lb} \times 1}{2 \text{ sq inches}} = 0.036 \text{ lb/sq in}$)

3. Total underground pressures
The total pressure underground is the lithostatic pressure and hydrostatic pressure added together since these both act together (with the atmospheric pressure added on top).

Table for water

Depth	Depth equivalent	Multiply figure above by:	Pressure using figures above
1 yard	Table to the floor	36	*0.036 x 36 = 1.30 lb/sq in*
100 yards	Short borehole	100	*130 lb/sq in*
1 mile	Longer borehole	17.6	*130 x 17.6 = 2288 lb/sq in*

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The back up
Title: Under pressure

Subtitle: Calculating the intense pressures underground

Topic: Measuring the pressure from a measured thickness of sand/water and using this in calculation to give an idea of the intense pressures underground.

Age range of pupils: 12-18 years

Time needed to complete activity: 20 minutes

Pupil learning outcomes: Pupils can:
- use an electronic balance;
- do arithmetical calculations;
- give an idea of the intensities of pressure
- expected at various underground depths.

Context:
This activity uses lab measurements of the force applied by different depths of sand and water to calculate their downward pressure and then uses these figures to extrapolate to likely pressures at crustal depths.

The activities (with both sand and water) begin by filling the cylinder to a fixed depth before the first weighing, since the bottom of a cylinder is curved and so the first depth of sand/water gives an unreliable measurement.

Actual lithostatic pressures would be greater than the calculations for loose sand suggest, because pressures from the rocks above (overburden pressures) cause compaction of sands which (together with cementation) forms them into sedimentary rocks; these are more dense than loose sands. The loose sand used in the calculations above had a relative density of1.8 whilst the relative density of sandstone is around 2.7, nearly 50% greater.

The deeper the rock, the more compressed and denser it becomes, increasing the pressure on the rocks beneath. Rocks deep in the crust are usually igneous and metamorphic rocks, which usually have even greater density than sedimentary rocks.

At every depth the material is in equilibrium because the pressure per unit area is exactly equal to the mass of material above unit area multiplied by the acceleration due to gravity

Following up the activity:
Follow this with the 'Water pressure – underground: demonstrating how hydrostatic pressure increases with depth' Earthlearningidea activity.

Underlying principles:
- The thicker the overlying rock, the greater the pressure on the rocks beneath.
- The deeper the water body, the greater the hydrostatic pressure.

Thinking skill development:
Calculation of the different figures for depth allows a pattern to be constructed, which is then bridged to the 'real world' of crustal depths.

Resource list:
- measuring cylinder (eg. 200 ml)
- pen to mark the measuring cylinder
- electronic balance
- ruler
- dry sand and water

Useful links:
http://www.nationalstemcentre.org.uk/elibrary/resource/1161/unit-14-who-s-for-a-hot-tight-squeeze-in-inner-space

Source: This activity is modified from an idea devised by David Thompson, and published in '*Who's for a hot, tight squeeze in inner space'*, Unit 14 of the Earth Science Teachers' Association's '*Science of the Earth'* series (1989) published by Geo Supplies, Ltd, Sheffield. David passed away recently and this Earthlearningidea is published in his memory.

We are most grateful for the help of Keele Education technicians Suzy Allen and Gwyn Jones in setting up and testing the apparatus, and to Martin Devon for his constructive comments.